JOURNAL OF AIRCRAFT
Vol. 47, No. 3, May–June 2010

Technical Notes

Effect of Jet-Exhaust Streams on Structure of Vortex Wakes

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DOI: 10.2514/1.47427

Nomenclature

A	=	wing planform area, ft^2, m^2
AR	=	b^2/A wing aspect ratio
b	=	wingspan, ft, m
b'	=	spanwise distance between vortex centers $\approx \pi b_g/4$, ft, m
C_L	=	Lift$/q_\infty S$
G_o	=	$\Gamma_o/b_o U_\infty$
Lift	=	$\rho_\infty U_\infty \lvert\Gamma_o\rvert b'$, lbs, N
q_∞	=	$\rho_\infty U_\infty^2/2$
S	=	planform area of wing, ft^2, m^2
T	=	$t\Gamma_o/b_o^2$
t	=	time, s
U_∞	=	velocity of wake-generating aircraft, ft/s, m/s
u, v, w	=	velocity components in x, y and z directions, ft/s, m/s
Wt	=	weight, lbs, N
X	=	x/b_o
x	=	distance in flight or longitudinal direction, ft, m
y, z	=	distance in lateral and vertical directions, ft, m
Γ	=	circulation bound in wing, ft^2/ sec, m^2/s
γ	=	$-\mathrm{d}\Gamma/\mathrm{d}y$
ρ	=	air density, slugs/ft^3, kg/m^3

Subscripts

ell	=	elliptical spanwise loading
g	=	wake-generating aircraft
o	=	reference quantity
pr	=	vortex pair
RTJ	=	R. T. Jones weight-optimized span loading
∞	=	freestream condition

Received 29 September 2009; revision received 21 January 2010; accepted for publication 26 January 2010.

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I. Introduction

THE study reported here supports efforts underway to increase airport capacity for landing and takeoff operations by use of closely spaced parallel runways [1–3]. For these operations, the aircraft fly in close formations (both laterally and along-trail) so that the following aircraft can avoid the hazard posed by the lateral spreading and movement of the vortex wakes of leading aircraft. The along-trail time intervals between aircraft flying in these formations will be on the order of seconds rather than minutes. To accommodate such close spacings, predictions of wake-spreading distances as a function of time must be available in about a second. In the past, research on the spreading of lift-generated wakes as a function of time assumed that engine power during approach and landing was at idle and negligible as far as wake-spreading rates were concerned [1–4]. For the envisioned formation approaches, engine power may be used during some portions of the approach. A computational method is suggested here as a tool for the prediction of the spreading of lift-generated wakes as a function of time when power is applied during closely spaced aircraft operations at airports.

The study reported explores the effect of energetic jet-exhaust streams from the engines on the aircraft on wake structure and spreading. An experimental resource for information is available from the condensation trails behind aircraft at cruise altitudes. Aircraft at cruise altitudes not only have energetic jet-engine-exhaust streams, but also often provide flow visualization of their lift-generated and exhaust-driven wakes at large distances (and long times) behind the wake-generating aircraft ‡. In addition, existing computational tools provide a variety of reliable numerical methods for the detailed history of the structure of the wakes of aircraft at cruise altitudes [5–14]. Several of these methods are so complete in their simulation that they account for the presence of energetic engine-exhaust streams, buoyancy of heated gases, wind shear, and thermal effects due to changes of state of water vapor. The numerical results obtained are in close agreement with the structure of wakes observed in condensation trails at cruise altitudes [9–16]. Unfortunately, these methods require a large amount of input data from the atmosphere and from the wake-generating aircraft. Consequently, those methods use computational times that exceed the short cycle times needed for safe and efficient compressed landing and takeoff operations at airports.

It was therefore decided to explore the use of more approximate, simple singularity-superposition methods to simulate wake structure as a function of time during the early part of the wake before the long-wave instability begins [17–21]. Point-vortex singularities are used to simulate the dynamics of the lift-generated vortex sheet shed by a wing. Point-source distributions around the periphery of each engine-exhaust port are used to simulate the growth in the volume of gases created by the combustion of fuel with oxygen to produce a large volume of heated nitrogen, carbon dioxide, and water vapor. The effective volume of exhaust-marked gases is also increased substantially by the mixing of exhaust products with ambient air. Because the computed results are two-dimensional and time-dependent, they simulate wake dynamics during the first part of their existence. The method is, however, not able to simulate the development of three-dimensional time-dependent dynamics in the

‡Condensation trails appear behind aircraft when they fly at cruise altitudes, because the atmosphere at those altitudes is often at temperatures below −40°F (or −40°C), and a relative humidity above 40%. These conditions cause the water vapor in engine-exhaust gases to condense and freeze into ice crystals before they have time to evaporate or sublime. Under those conditions, exhaust condensate becomes a fluid marker of long duration that makes it possible to observe the dynamics of lift-generated vortex wakes over a wide range of conditions.

wake, such as the long-wave instability. The computational method also cannot be used to estimate when the airspace for a given runway or runway system is safe to recycle. It is interesting to note that engine thrust does measurably reduce the intensity of wake-induced rolling moments, but it does not alleviate wakes enough to render them harmless [21,22].

The purpose of the present study is to develop and test a simple and rapid method for estimation of how jet-engine-exhaust streams spread as a function of time. Of particular interest for wake-avoidance purposes is how the wakes of aircraft flying at cruise altitudes differ during their first half-minute of existence from those that trail from aircraft under low-power conditions. For this purpose, theoretical and observational information is used to test the prediction capability and understanding of differences in the structure of lift-generated wakes of aircraft as brought about by energetic jet-exhaust streams from the engines on the aircraft. Although the development carried out is not based on existing powerful computational tools described in the literature [7–14], it is recommended that these previously developed methods for wake structure at cruise altitudes be retained as backup support for possible diagnostic computations if problems arise during use of the simpler analysis tools now planned for service.

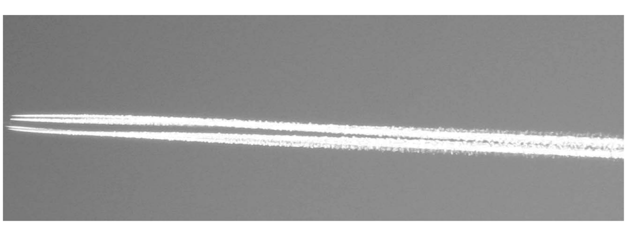
II. Photographs of Condensation Trails

The computational method to be tested is assumed to apply from the time that the wake is generated just behind the aircraft, as shown on the left side in Fig. 1a, through the condensation wrapping phase shown in Fig. 1b. It is assumed that the wake is dominated by two-dimensional effects, and that three-dimensional time-dependent dynamics do not become dominant until the initiation of the long-wave instability of a vortex pair begins (Fig. 1b) and goes to completion in Fig. 1c. The view of the wake from the ground was at an angle of 20–30 deg from the vertical. Because of the large distance between the wake and the observer and the odd angle of observation, it is difficult to determine which features of the wake are in side view and which are from below the bulk of the wake condensate. For example, in Fig. 1a, the exhaust from the two jet engines on the starboard side of the aircraft are seen edge on as if only one engine existed. In contrast, the exhaust streams from the two on the port side are observed in a slanted plan view that displays both exhaust streams in the early part of wake history. As the wake ages, the velocity field of the lift-generated vortex sheet rotates the planes of the two jet plumes on each side of the aircraft so that they appear to grow and shrink in diameter, but are simply rotating about each other. In addition, the swirling velocities of the trailing vortex system move the mixture of exhaust gases, ambient air, and condensate from the lower part of the wake around the sides of the wake to form a thick layer of condensate and ambient air at the top of the wake (Fig. 1b). During that time, the lift-generated vortex pair of high-speed cores descends to the bottom of the bulk of the wake.

As time progresses, the depth of the layer on top of the wake persists and reforms itself into several layers that appear to have a vortical basis because of the opaqueness striations in the structure of the upper part of the wake (Fig. 1c). The across-wake vertical striations in the wake are no doubt associated with the arrays of exhaust shear-layer vortices that were generated at the interface between the jet streams and ambient air. The downward sloping clumps of condensate in Fig. 1c are manifestations of the later stages of the long-wave instability of a vortex pair [6,8].

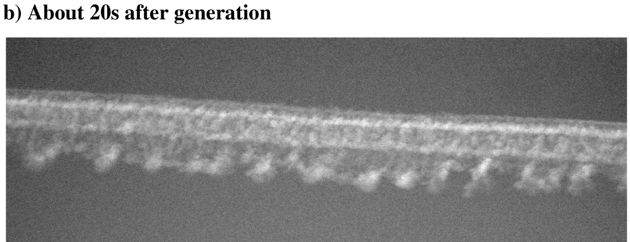
The array of shear-layer vortices that surround the jet streams along their length are first seen as periodic roughness on the outer surfaces of the jet streams (Fig. 1a), and then as circumferential striations mixed with exhaust condensate striations (upper part of Figs. 1b and 1c). The exhaust shear-layer vortex array is not simulated by the primitive singularity method used to produce the wake figures in the following section. Also not simulated by the numerical method being used are the waves that begin to form on the vortex pair at the bottom of the wake (Fig. 1b). As will be shown in figures to follow, the singularity method does simulate the movement of the vortex pair from the center of the wake when just behind the aircraft downward to the bottom of the condensation trail.

The clumps of condensation at the bottom of the formation shown in Fig. 1c are obviously the residue from the long-wave instability. At that time, the wake appears to a have an upper and a lower part. Such a configuration suggests that the wake has formed into two separate wake systems that remain close to each other. A confirmation of the separation of condensation trails into two nearby parts that differ in character is shown in Fig. 2. The picture was taken from an aircraft flying beside and into the wake of another aircraft to make measurements of wake velocities. The objective of the flight-test program was to evaluate potential hazards to aircraft that might accidentally penetrate a lift-generated wake. At the time of the picture, the lower part consists of only a vortex pair that is going through the early parts of the long-wave instability, but has not yet formed closed loops of vortex filaments. The upper part of the wake does not appear in this picture to have any organized structure. Other pictures of the upper part indicate that it contains large eddies [15] that resemble a highly turbulent flowfield. Clearly defined along-trail vortex structures are not apparent in the upper part of the wake shown in Fig. 2. In the photograph, the formation of two distinct regions of wake fluid indicate that the wake has, in this case, divided into two



a) Beginning of condensation trail and spread of wake

b) About 20s after generation



c) About one minute after generation

Fig. 1 Diagonal views of condensation trail from ground indicating wake structure of aircraft with four jet engines.

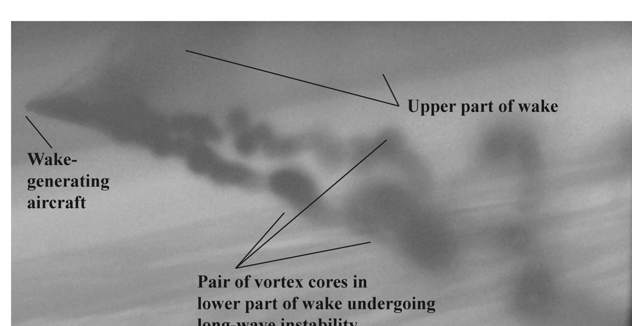


Fig. 2 Photograph taken from nearby aircraft of the condensation trail of a transport aircraft at cruise altitude.

parts that have different structure and continue to aerodynamically communicate with each other. The upper part appears to be a cloud of condensate without any organized structure or presence of an organized vortex pair. Other photographs of condensation trails [15] indicate large-scale swirling motions that protrude as a part of the upper surface of the condensation trail. The upper region is sometimes referred to as the jet-wake part of the condensation trail, because it appears to consist primarily of engine exhaust. The lower region is then described as the vortex part of the wake, because a pair of vortex cores undergoing the long-wave instability is the only prominent feature in that part of the wake.

III. Trefftz-Plane Analysis of Wake Dynamics

A. Overview of Method

The computational method used to compute the time-dependent dynamics of lift-generated vortex wakes with and without the presence of jet-exhaust gases is made by use of point-vortex and point-source distributions in the so-called Trefftz-plane [17–21]. The simulations begin just behind the wake-generating wing where the initial location and structure of the wake are known. The atmosphere is assumed to be stationary and uniform. The Trefftz-plane approximation simplifies the computations by use of the assumption that variations in wake-induced velocities in the first part of wake history are most important in the vertical and lateral directions, and negligible in the flight direction. Therefore, variations of the streamwise components of wake velocities from the freestream velocity are ignored. The roll-up of vortex sheets can then be treated as a time-dependent, across-stream problem. As a consequence of the assumptions made, the technique cannot represent three-dimensional instabilities like the long-wave instability of a vortex pair [6,8].

B. Spanwise Loadings Considered

Because the spanwise loadings for aircraft that generate condensation trails are not available, two idealized span loadings were chosen for study (Fig. 3) to find out if wake dynamics is sensitive to the design of the spanwise distribution of lift. The first span loading chosen is elliptical span loading, derived by Glauert [23] and Munk [24] as the span loading that corresponds to a uniform downwash across the wing span and has the least lift-induced drag for a given lift and wing span. An elliptically loaded wing has a spanwise loading given by

$$\Gamma_{\mathrm{ell}}(y)/\Gamma_o = (4/\pi)[1-(2y/b_o)^2]^{1/2} \tag{1}$$

where $\Gamma_{\mathrm{ell}}(y)$ is the spanwise distribution of circulation bound in the wing to represent the lift on an elliptically loaded wing of span b_o that carries the same total lift ($=\rho|\Gamma_o|b_o U_\infty$) as a wing with uniform loading Γ_o and a span of b_o.

The second span loading chosen for study is the span loading designed by Jones [25] as the span loading that has the least lift-induced drag for a given lift and given minimum wing-root bending moment (i.e., least structural weight). The Jones-optimized design, RTJOPT, is a combination of triangular and elliptical downwash distributions along the span of the wing. Because the Jones design has more restraints than the Munk design, the span of the wing must also be specified to fix the amount of lift and efficiency increase. The ratio of the spans of the two designs used in this study is given by $b_{\mathrm{RTJ}}/b_o = 1.15$. That value is the particular span ratio recommended by Jones [25] as the most practical for weight-optimized designs. The span loading is written in closed form as

$$\begin{aligned}\Gamma_{\mathrm{RTJ}}(y)/\Gamma_o &= [(12/\pi + 6Yp)/(b_{\mathrm{RTJ}}/b_o)][1-(2y/b_{\mathrm{RTJ}})^2]^{1/2}\\ &+ \{(18Yp - 24/\pi)(4y^2/b_{\mathrm{RTJ}}^2)\,\ell n[(b_{\mathrm{RTJ}}/2y)\\ &+ ((b_{\mathrm{RTJ}}/2y)^2]^{1/2}\}/(b_{\mathrm{RTJ}}/2y)\end{aligned} \tag{2}$$

where $\Gamma_{\mathrm{RTJ}}(y)$ is the spanwise distribution of bound circulation in the wing. The quantity Yp is defined as $Yp = 4b_o/(3\pi b_{\mathrm{RTJ}})$.

C. Computational Method

The idealized structures of the singularities to be used in the computations have excessive velocity distributions near their centers, where infinite velocities occur. If the singularities are used in their idealized form, the conservation relationships provided by vortex invariants are preserved. However, large unrealistic random motions of individual singularities then frequently come about when the centers of singularities come near each other, even though very small time steps are used in the computations [17–21]. The disorderly nature caused by use of idealized singularities make comparisons difficult because the wake structures are then not orderly, as expected for smoothly changing span loadings. Lack of order is especially apparent in the center regions of the rolled-up core regions of the vortices.

Therefore, to facilitate comparisons by bringing about smoothly changing dynamics within the wake structures to be studied, both point-vortex and point-source singularities were given soft centers. Soft centers were achieved by changing them from a $1/r$ variation, to a Rankine type, or r, variation. The size of the center modification needs to be large enough so that smooth movement of the vortices and sources occurs in all the problems being treated. By use of a trial and error process, it was found that the core structure of the singularities had to be changed over a radial distance of four vortex spacings. For comparison, 40 vortices are used to represent the span

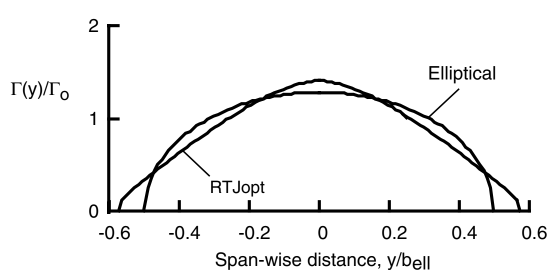
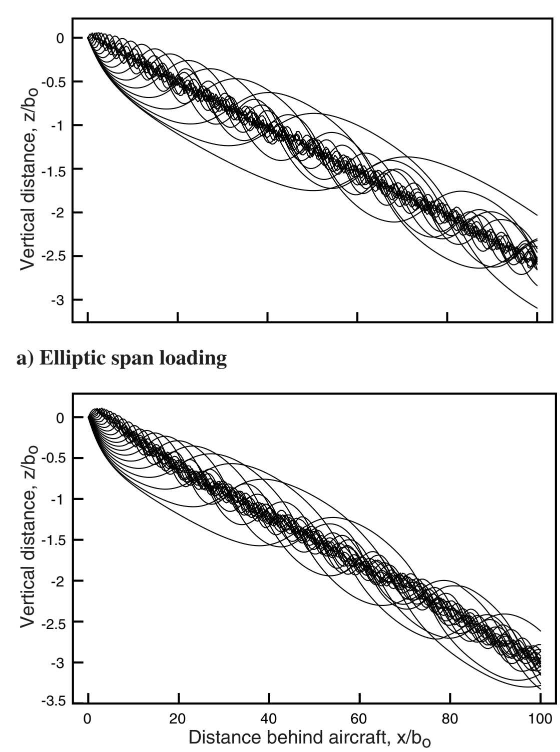


Fig. 3 Span loadings on isolated wings to be used as basis for study of wakes. Total lift by each lifting distribution is the same, so that Lift = $\rho U_\infty|\Gamma_o|b_o$.



a) Elliptic span loading

b) RTJ span loading

Fig. 4 Side view of the long-term roll-up process of the vortex sheets shed by the two spanwise loadings on a wing.

loading on the elliptically loaded wing, and 46 are used for the wing with the Jones loading. The approximations used at and near the centers of the singularities then provided orderly motion of vortex and source singularities so that it was quite straightforward to determine when different wake dynamics occurred due to configuration changes. However, as is well known [17–21], the change to soft centers causes the vortex invariants to change with time, indicating that a systematic error is being introduced into the vortex system. It is also noted that addition of source distributions to the flowfield also causes the vortex invariants to change with time [17]. It was decided to accept these types of conceptual errors because the technique used did produce orderly formations of the wakes being studied. The results presented are therefore not mathematically exact, but do provide realistic representations of wake development. The most obvious misrepresentation caused by soft centers is in the wing-tip region of the wakes.

The initial spanwise distribution of circulation in the vortex sheet shed by the wing at the moment the aircraft passes through a vertical plane at the trailing edge of the wing yields the initial distribution of circulation in the vortex sheet as

$$\gamma(y,0) = -\mathrm{d}\Gamma(y,0)/\mathrm{d}y \tag{3}$$

where $\Gamma(y,0)$ is the magnitude of the bound circulation in the wing as a function of spanwise distance.

As indicated in figures to follow, the sources are initially assumed to be located on the periphery of the initial outer edge of the engine-exhaust plumes as the beginning structure of the jet streams. The timewise development of the vortex sheet in the plane where the computations are being made moves away from the trailing edge of the wake-generating aircraft at the flight velocity U_∞, so that it remains with the same body of fluid during the event. In this way, the two-dimensional time-dependent computations produce a representation of the three-dimensional structure of the wake.

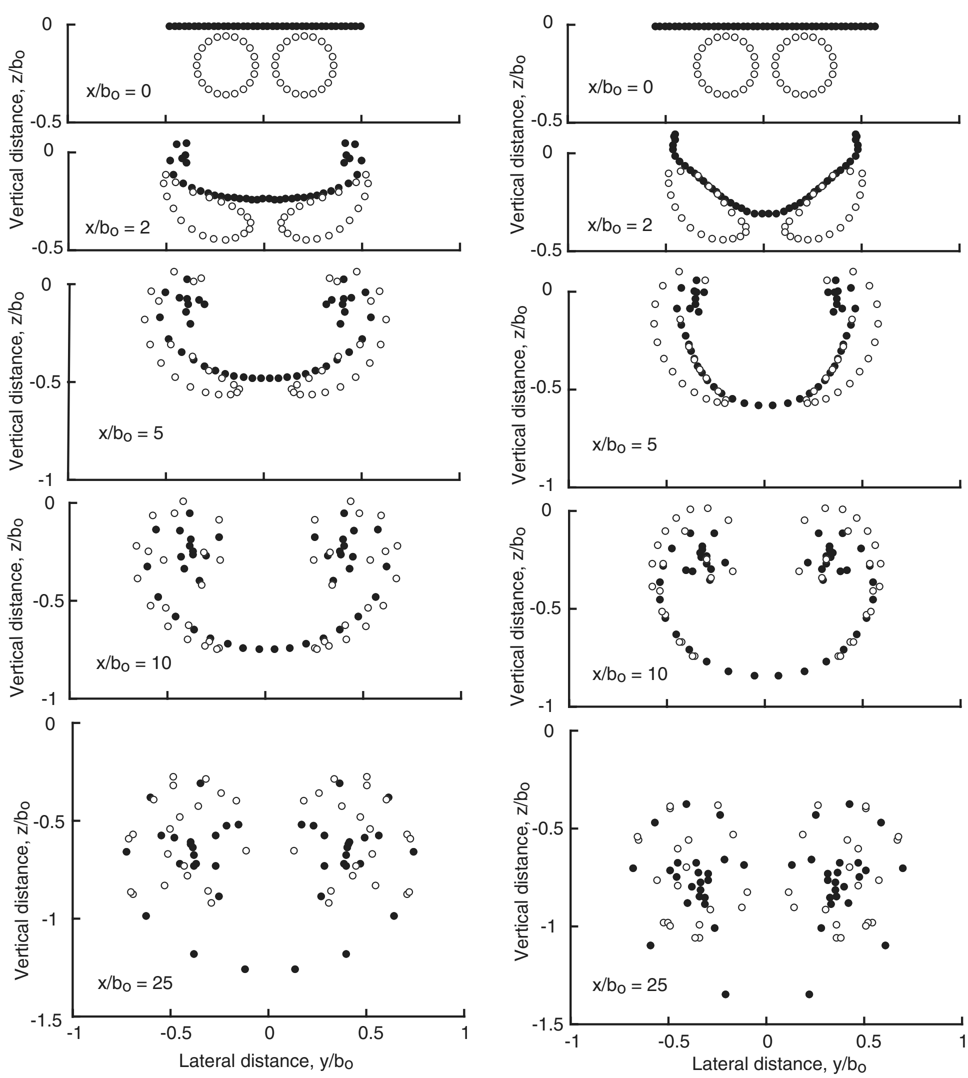
The equations used are made dimensionless by dividing the lateral v_i and vertical w_i velocity components by the flight velocity U_∞ of the wing through the atmosphere. In the computations, the time is made dimensionless by a grouping of parameters as $T = t\Gamma_o/b_o^2$. The various quantities are also related to one another through the lift on the wing as

$$\mathrm{Lift} = \rho U_\infty |\Gamma_o| b_o \tag{4}$$

The time parameter is also related to the self-induced downward velocity of the vortex wake given by

$$w_{\mathrm{pr}} = |\Gamma_o|/2\pi b'_o \tag{5}$$

where $b'_o = b_o\pi/4$ and $\Gamma_o/b_oU_\infty = 2C_{Lg}/\pi AR_g$. Because $C_{Lg} \approx 1.0$ during cruise flight and $AR_g \approx 7$ for subsonic transports, the distance in span lengths, x/b_o behind the wake-generating aircraft is related to the computational time by



a) Elliptic span loading

b) R. T. Jones span loading

Fig. 5 End views of locations of point vortices (●) and passive fluid markers (\circ) at several times during the history of wake dynamics out to 25 span lengths.

$$X(T) = x(t)/b_o = tU_\infty/b_o = T/(\Gamma_o/b_oU_\infty) = T/G_o \quad (6)$$

where $G_o = \Gamma_o/b_oU_\infty$ is around 0.1–0.2. In the figures to follow, G_o will be assumed to be 0.2 for simplicity and for compactness. The distance behind the wake-generating aircraft is then related to the dimensionless time by $x/b_o \approx 10T$.

V. Computational Results

A. Side Views of Wake Dynamics Without Exhaust Plumes

The side views presented in Fig. 4 were computed by the numerical method discussed previously to show the paths traveled by the point vortices as a function of downstream distance, as predicted for classical vortex wakes [4]; i.e., no sources are present. The paths predicted for the point vortices approximate vortex filaments as they trail downstream from the wing for about 100 span lengths of travel ($\approx$0.5 min for an aircraft at a subsonic cruising velocity of about 800 ft/s and a span of 200 ft). Figure 4 displays the changes in wake structure as a function of time during the roll-up process along with the smaller changes that occur after roll up at larger distances behind the wing. The figures illustrate how the individual vortex elements orbit about a central core region where the centroids of vorticity (or circulation) are located. The filament motions indicate that multiple wake regions do not form for these two configurations. Such a result contrasts with the structures observed in Figs. 1c and 2, probably because engine-exhaust plumes are not present. The motion of the two vortex sheets differs most at the beginning of the roll-up process where the time at which the outermost vortices take longer to be rolled into the vortex structure for elliptic loading than for the RTJ loading. Also, the paths of the vortices for the R. T. Jones (RTJ) loading appear to be more compact than those for elliptic loading. The larger circulation content and the more inboard loading of the RTJ vortex sheet cause it to wrap more tightly and to descend a bit more rapidly than the vortex sheet shed by an elliptic-loaded wing.

B. End Views of Motion of Point Vortices with Markers

To better understand the mechanisms that cause condensation trails to grow in cross-sectional size from about one span to over five

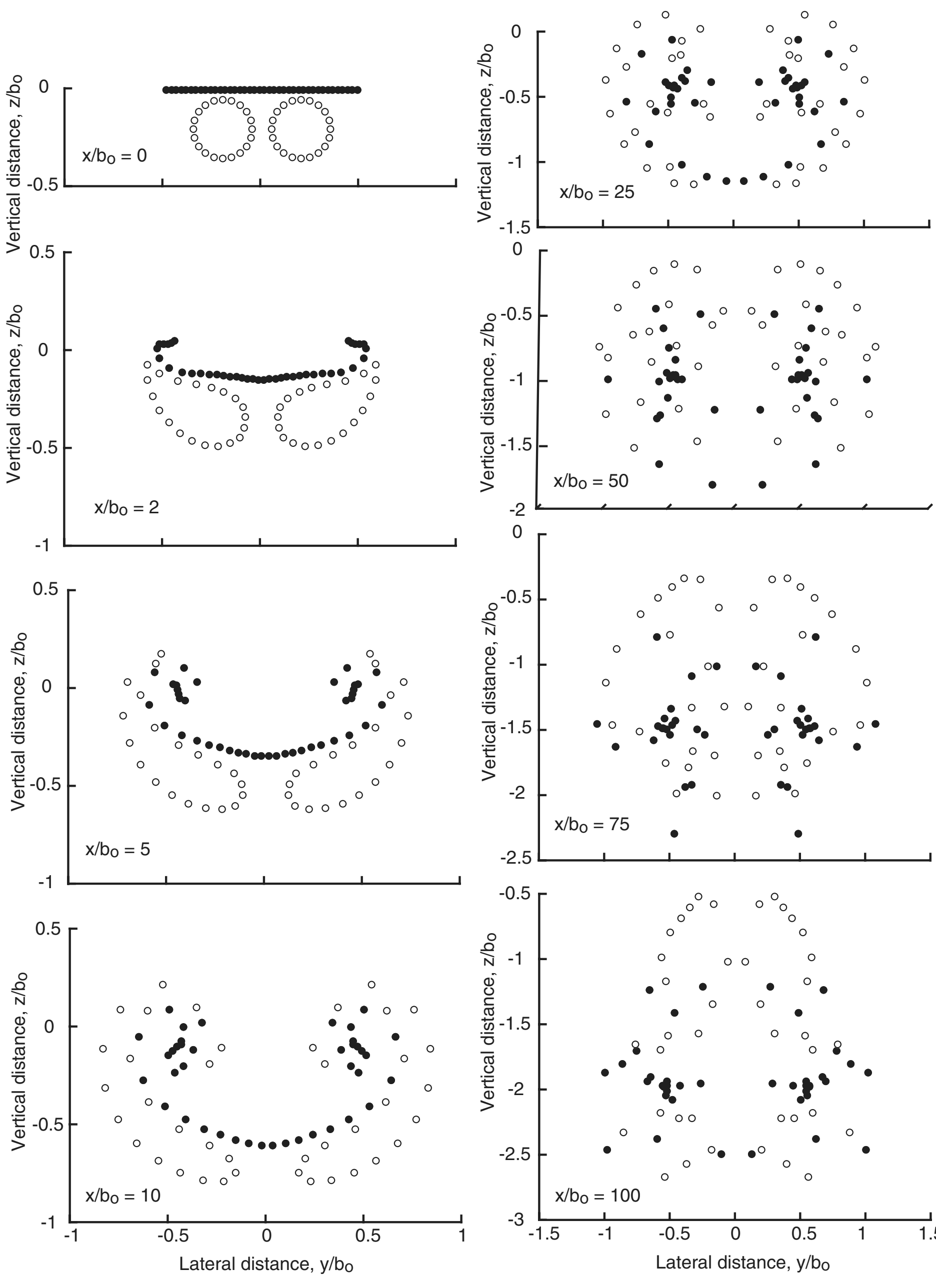


Fig. 6 Dynamics predicted for lift-generated wakes shed by elliptically loaded wing as represented by point vortices and sources to simulate effect of exhaust products and mixing with the atmosphere on wake structure.

spans over relatively short periods of time, the wake dynamics for the two span-load configurations were calculated with the open circles serving as fluid markers rather than sources. The markers simply float with the local velocity field to show how ambient fluid in the underwing region would mix with the vortex wake shed by the aircraft. The dynamics in the wake and ambient fluid are shown in rear view at several different early times in the development of the wake (Fig. 5). The results indicate how the ambient fluid at the periphery of the engine-exhaust openings is enveloped by the roll-up process brought about by the vortex part of the wake, in the same way that classical roll up occurs [4]. The open circles show how the fluid under the wing, where engines would be located, mixes with the vortex part of the wake.

C. Rear Views of Motion of Point Vortices with Sources

Enlargement of the cross-sectional size of the engine-exhaust stream by heating, by production of chemical-combustion products and by mixing with ambient air is now included by addition of point-source distributions around the periphery of what are assumed to be two engine-exhaust openings (Figs. 6 and 7). Each source has a dimensionless strength of 0.05 as chosen by a trial and error process to be a midrange value for the cases computed. Smaller values had little effect on wake structure, and larger values excessively dominated wake dynamics. It was found that the value chosen for the summation of all of the strengths of the sources is the same dimensionless magnitude as the total strengths of point vortices on each side of the wake.

The theoretical simulations presented in Figs. 6 and 7 indicate that the fluid added to the wakes by exhaust gases (open circles) are moved to the top of the wake by the action of the swirling flowfields brought about by the point-vortex distributions. In fact, the sources generally remain near the altitude where they began. In comparison, the vortex part of the wake moves downward under the self-induced velocity field of the vortices, so that as the wake ages it becomes composed of an upper (mostly condensation) part and a lower (mostly vortex) part, as observed at cruise altitudes (Figs. 1 and 2). The two parts separate slowly with time, but remain in communication with each other [15,16]. Also, the point vortices appear to gather into several groups of pairs, rather than a single group, as occurs in the classical case. The amount of separation between the two parts of the wake as shown in Figs. 1 and 2 are slightly different for the two span loadings because the total circulation and wake-wrapping power of the RTJ loading is greater than the wrapping power of elliptic loading.

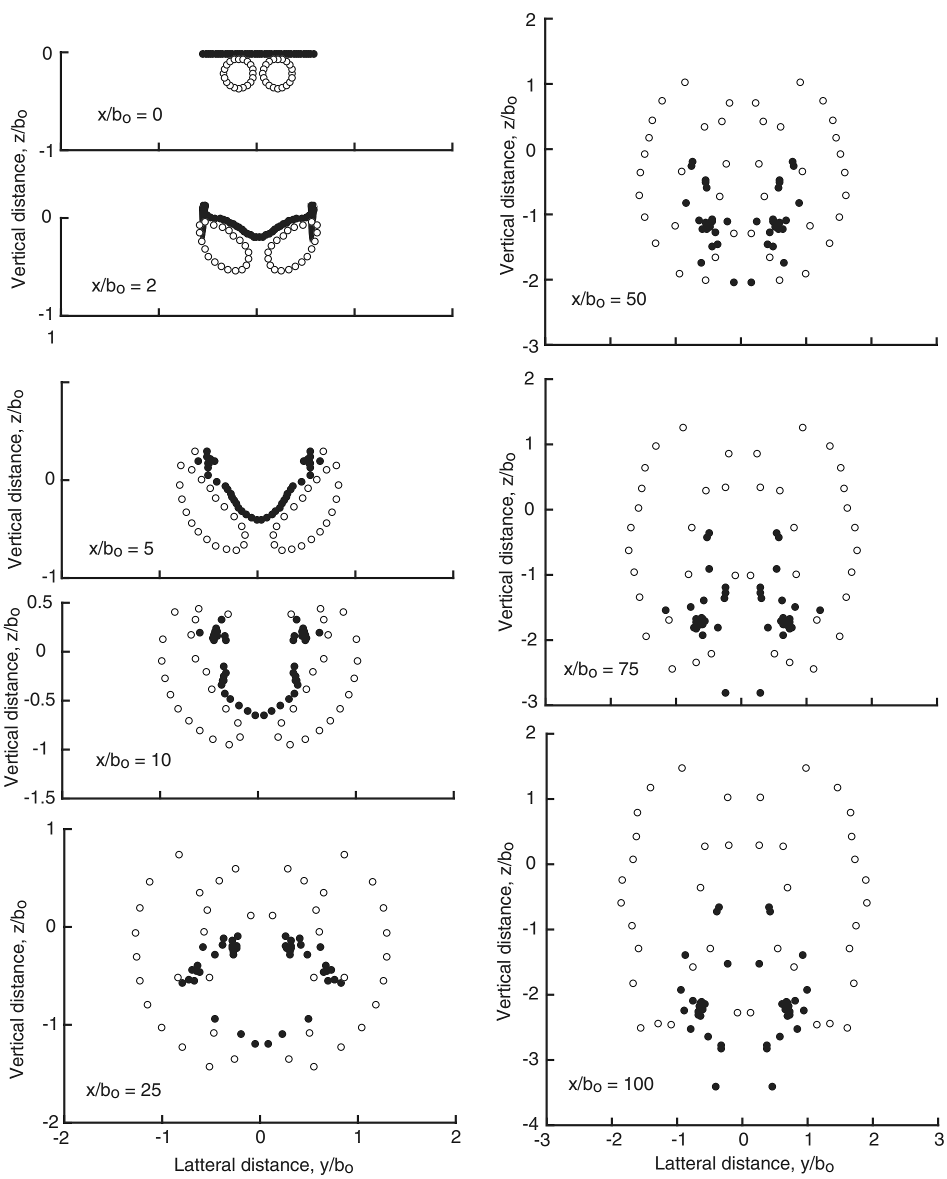


Fig. 7 Dynamics predicted for lift-generated wakes shed by wing loaded according to R. J. Jones' design for minimum weight, as represented by point vortices and sources to simulate effect of exhaust products and turbulent mixing with the atmosphere on wake structure.

V. Discussion of Results

As shown in Figs. 6 and 7, the point sources first begin to mix with the point vortices but then separate into an upper wake region as exhibited in condensation trails (Figs. 1 and 2). In both cases, the point sources move from the circular distributions from around the periphery of the exhaust streams outboard, around the wing tips, and then to the upper part of the wake. The separation of wake components into an upper and lower part comes about because the emission type of flowfield created by the point sources does not induce a swirling motion in the flowfield, but tends to push the singularities away from each other. The point-vortex pairs tend to move downward under their self-induced velocity field so that the point sources are left behind. It is noted that when the circular symbols represented fluid markers and not point sources, the wakes did not separate into parts (Fig. 5) but rolled up into a single pair of singularities.

The RTJ loading does not mix sources and vortices as intimately as the elliptic-loaded case, because elliptic loading has more vorticity concentrated at its wing tips than the RTJ loading cases. RTJ loadings have a larger total circulation that is more evenly distributed across the span of the wing than elliptic loading. The distribution of circulation along the wing causes a larger number of sources to accumulate at the top of the wake in the RTJ case. This separation trend persists as time increases so that, at $x/b_o = 100$, most of the sources are located above the vortices in the RTJ case, and are more mixed with vortices in the elliptically loaded case. In both cases, the presence of the point sources tends to divide the single group of point vortices shed by the wing into several layers of point-vortex groups, so that a vertically layered appearance is generated in the wake. The breadth and depth of the wakes shed by both span loadings are more than doubled when energetic jet-exhaust streams are added to the flowfield. The results also indicate that cross-sectional size of the wake is significantly enlarged by jet-exhaust streams, making it necessary to make an adjustment in rates of wake spreading for wake-avoidance computations. The enhancement of wake-spreading rates by jet-exhaust streams again calls attention to the fact that compact operations at airports need to be carried out on a single plane, so that aircraft do not fly above or below the flight paths of other aircraft [26].

VI. Conclusions

It is concluded that the Trefftz-plane method used here to facilitate computation of lift-generated wakes is probably adequate for estimates of wake spreading with and without jet-exhaust streams. The computed simulations of lift-generated vortex wakes with energetic jet-exhaust streams displayed in the forgoing figures indicate the same wake patterns observed in condensation trails at cruise altitudes. Because the method is a two-dimensional time-dependent computation, it can only be used to analyze the wake from the time that the wake was generated to the time when the long-wave instability begins to be initiated. At greater downstream distances more complete wake representations described in the literature are required. Guidelines derived previously for the motion and spread of vortex wakes for aircraft on approach to an airport are probably reliable as derived for approach and landing operations at airports, and need not be modified if power is not applied. However, acceptable accuracy of wake-spreading estimates when power is applied for approach adjustment, and for takeoff, climb, and cruise operations, will require that the effect of jet-exhaust streams be included.

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